International Journal of Modern Manufacturing Technologies
ISSN 2067–3604, Vol. II, No. 2 / 2010

NEW GENERAL CALCULUS METHOD FOR SWEPT-BACK AND FRONT ANGLE OF WING

Gheorghiță Toncu[1], Virgil Stanciu [1] & Ionel Preda [2]

[1]"POLITEHNICA" University from Bucharest-Romania, Department of Aerospace Engineering, Gh. Polizu Street, No. 1, Sect. 1, CP 011061 Bucuresti, Romania
[2] Navy Academy "Mircea cel Bătrân" of Constanta, Department of Naval Wepons, Fulgerului Street, No. 1, CP 900218, Constanta, Romania

Corresponding author: Gheorghiță Toncu, tg@canals.ro

Abstract: The paper shows a new method for determining the sweepback and forward angles of the leading edge and trailing edge of the wings, empennages or ailerons, taking into account the critical point and the critical speed on an aerodynamic profile for the compressible subsonic or the transonic domain, for several NACA symmetrical profiles, according to NACA Report No. 824 - Summary of Airfoil Data. Also, a comparison of efficiency is offered for some profiles adapted to high speed fligh.
The research becomes useful for future design and developments of cruise missiles and airplanes aerodynamic surfaces, considering the relationships between the leading and trailing edges and the other geometric elements of them (incluing the derived ones).
Key words: aerodynamic, sweepback angle, Mach, section, surface, virtual.

Nomenclature

b - span; c_0 - root chord; c_e - tip chord; D - drug force; F_Σ - total lifting force (initial lifting force); F_{ar} - wing lift; F_{cp} - body lift; F_{ap} - command empennage lift force; F_{aa} - aileron command lift force; F_{dd} - directional lift force; G - weight; L - lifting force; M - Mach number; r - taper ratio; S_{da} - aerodynamic surface; S_{dd} - command surface; S_v - virtual surface; x, y, z - reference frame, co-ordinate; T - thrust force; V - velocity; Y - lateral force.

Greek leters

Ψ - rolling angle; Θ - pitch angle; Φ - azimuth angle; χ - sweepback angle; ε - airfoil section thickness; φ - semiplane angle.

1. INTRODUCTION

The up-to-date concept in the flying machines design asks first to establish the aerodynamic configuration, which is a step straight connected to the operational demands and utility needs of the machine. That means the "hardware" programming of the flying machine, which is straight connected to the form of the body, but to the equipment (control, propulsion and manoeuvrability) too, as unity (AGARD, 1995). The most used aerodynamic configurations are shown in Fig.1 (Anderson, 2001). The most important elements considered to be direct connected to the aerodynamic configuration of the flying machine are: stability, manoeuvrability and controlability (Cook, 2007), (Dingle & Tooley, 2005), (Torenbeek, 1976).
So, the main problem of flight is connected to the stability of the flying machine (Etkin, 1959), (Etkin, 1972). Contrary, the knowledge of instable behavior of the flying machine in flight maneouvre becomes important for avoiding the flight accidents and overall military consideration. The essential condition for a stable flight, results from the equilibrium of the forces acting on the flying machine (Hull, 2007), (Jenkinson & Marchman, 2003), (Kroo, 2001).
There are many possibilities to arrange supporting surfaces - wings, empennages or ailerons (Fig.2). Usually, the symmetric monoplane arrangement with or without dihedral angle is used (Fig.2a…d). The general case of the aerodynamic surfaces disposal is considered for semiplans symetrical disposed and having the angle $\varphi = \pi/4$ between a semiplane and the main plane of the flying machine - longitudinal or horizontal (Fig.2e); in some references are studied the asymetric arrangements too ($\varphi \neq \pi/4$, fig. 2f).
The form of aerodynamic surfaces on plain (Fig.3) is first based on the flying security through assuring the necessary upward and command and control forces on the flying machine. Additional, the aerodynamic interferences and turbulence reductions are intended (Kuethe & Chuen-Yen, 1976), (Krasnov, 1985).
At the same time, in flying machines design vertical and horizontal empennages and ailerons are used (Fig.4). Those have essential contribution to provide the lifting force and command of flying machine. Their forms and disposals are similar to those of the wings relative to the aspect ratio.

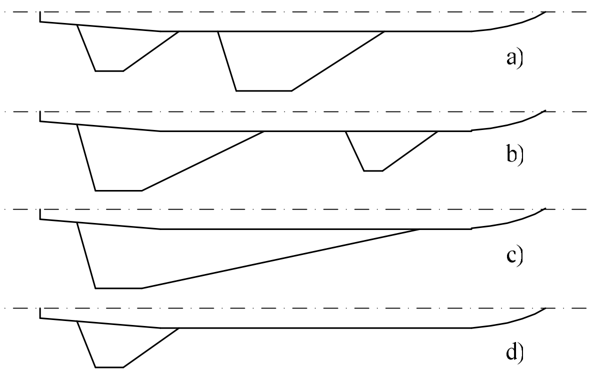

Fig.1. Aerodynamic configurations: a) normal; b) reversed (canard); c) without empennages; d) without wings

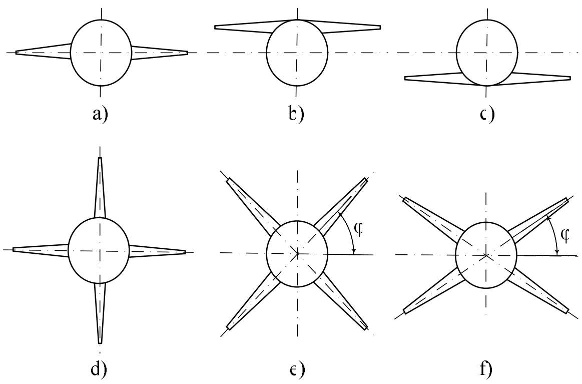

Fig.2. Aerodynamic surfaces arrangement: a÷c - monoplain (normal); d - on „+”; e - on „X”; f - variant of „X” (φ’ ≠ φ)

The main condition for a stable flight follows from the equality of the forces acting on flying machine:

- vertical: $L = G$ (1)
- longitudinal: $D = T$ (2)
- transversal: $Y = 0$ (3)

Taking into account the stability conditions for flying machines, it can be written (Milne-Thomson, 1966):

- lifting surface, from Rel. (1):

$$S_{xy} = \frac{2G}{c_L \rho V^2} \quad (4)$$

- drug surface, from Rel. (2):

$$S_{yz} = \frac{2T}{c_D \rho V^2} \quad (5)$$

But, from Rel. (3) it follows the fact that to obtain the stability of flying machine is necessary to have a transversal velocity equal to zero, so it will result the lateral drift for:

$$S_{zx} = \frac{Y}{c_Y \rho V^2} \quad (6)$$

It is necessary to mention that the aerodynamic control surfaces (Fig.5), as function of rotational axis position and the ratio between active surface into the flow and the aerodynamic surface, are:

- corrected (compensated): $S_{dd}/S_{da} = 1$;
- semicorrected (semicompensated): $S_{dd}/S_{da} < 1$;
- noncorrected (noncompensated): S_{dd}/S_{da} □ 1;

function of their position on considered aerodynamic surface they are:

- coincident = compensated (plane rotative) (Fig. 5a);
- ultimate = corrected, semicorrected (Fig. 5b, 5c);
- included (Fig. 5d).

In technical studies are mentioned other aerodynamic command surfaces too: control installed in jet stream and roleron. Adding the gasodynamic control devices the full image of command and control devices used for flying machines appears (Nielsen, 1960).

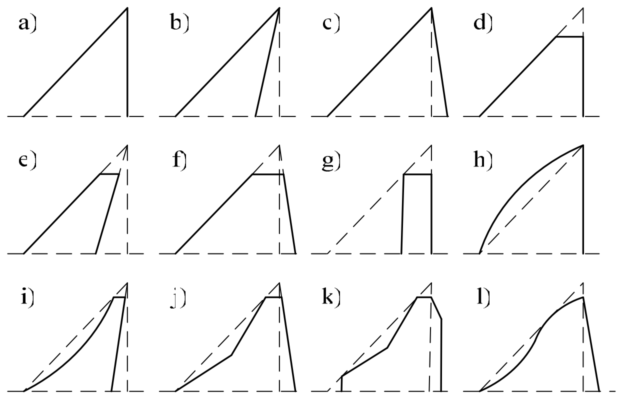

Fig.3. Plain forms for aerodynamic surfaces: a÷c – triangular; d÷f - trapezoidal; g - rectangular; h - gothic; i - sickle; j - two sweep-back angles; k - many sweepback angles; l - “S” form

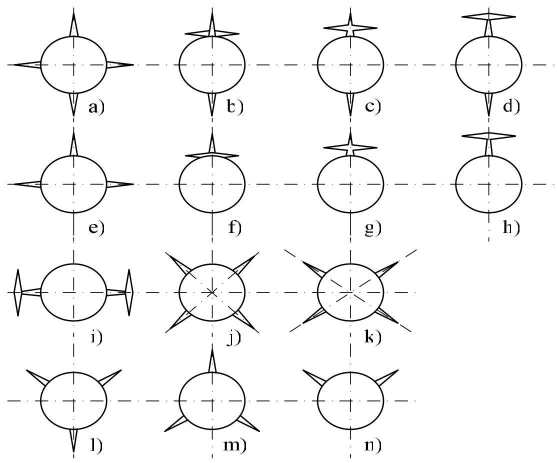

Fig.4. Empennages scheme: a - „+” form (normal); b÷i – mono-plains; i) – on „X” form; j÷m - variants of „X” form

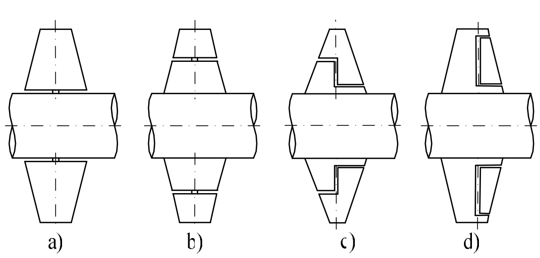

Fig.5. Command surfaces: a) compensate; b) ultimate; c) semicompensate; d) noncompensate (of trailing edge)

2. CONSIDERATIONS REGARDING THE VIRTUAL MODEL

On flying machine in aerodynamic evolution act the aerodynamic forces and moments but the thrust and weight too. The essential condition for a stable flight follows from the equilibrium of the forces acting on flying machine (Smetana, 1997), (Siouris, 2004).
For evaluating the aerodynamic behavior of flying machine is necessary to consider the geometric aerodynamic configuration of that machine because it determines the aerodynamic forces and moments. For a movement decomposed along principal axis of flying machine, these forces and moments can be synthetised as follows:

$$F_x = c_x \frac{\rho}{2} V_x^2 \, S_v\big|_{yz}$$
$$F_y = c_y \frac{\rho}{2} V_y^2 \, S_v\big|_{zx} \qquad (7)$$
$$F_z = c_z \frac{\rho}{2} V_z^2 \, S_v\big|_{xy}$$

Depending on aerodynamic configurations of flying machine, considering the modules of lifting forces acting on machine of form (1) it will follow:

- normal: $F_\Sigma = F_{ar} + F_{cp} - F_{ap}$;

- reverse: $F_\Sigma = F_{ar} + F_{cp} + F_{aa}$;

- without empennages: $F_\Sigma = F_{ar} + F_{cp} - F_{dd}$;

- without wings: $F_\Sigma = F_{cp} - F_{ap}$.

To determine the magnitude of aerodynamic surfaces that correspond considered aerodynamic forces, first will be analise only the general case of the four semi-planes (Fig. 2e; fig. 4j) and lifting forces that correspond to every semi-plane. Because lifting forces act in the same way on the surfaces, by projecting the semi-plane on horizontal and vertical planes it will follow:

$$\begin{aligned} S_v &= 4 \cdot S_{sp} \cos(\varphi) = \\ &= 4 \cdot S_{sp} \frac{\sqrt{2}}{2} = \\ &= 2\sqrt{2} \cdot S_{sp} \quad \text{for } \varphi = \pi/4 \end{aligned} \qquad (8)$$

By rotating the body around longitudinal axis *x*, the lifting force of the two semi-planes will be reduce as following projections:

$$S_v = 2 \cdot S_{sp} \cos(\varphi - \psi) + 2 \cdot S_{sp} \cos(\varphi + \psi) \qquad (9)$$

Where from, for $\psi \underset{\lim}{\rightarrow} \pi/4$, will be obtained the particular case of the monoplane surface:

$$S_v = 2 \cdot S_{sp} \qquad (10)$$

Also, it can be seen that rel. (8)–(10) are still valuable in the case of rotation of flying machine around vertical Oz axis. In the case of the rotation around lateral axis Oy a different projection appears.
The complexity of aerodynamic form which results in this case will be analised through a particular and different concenpt which is not studied in references.
Introducting the concept of *virtual surface*, that means a surface which does not exist in reality, but which *is obtained by projecting the considered mean (average) surfaces of flying machine*, will permit to obtaine the equivalent surface for specific aerodynamic calculus.
It can be seen that, in comparison with the surfaces of initial projections, the surfaces of projections as functions of simultaneous rotation angles of the body around the three referential axes becomes:

$$\begin{aligned} S_V\big|_{yz} &= S_{yz}\cos\Theta\cos\Psi + S_{zx}\left(\sin\Phi\sin\Theta\cos\Psi - \cos\Phi\sin\Psi\right) + \\ &\quad + S_{xy}\left(\cos\Phi\sin\Theta\cos\Psi + \sin\Phi\sin\Psi\right); \\ S_V\big|_{zx} &= S_{yz}\cos\Theta\sin\Psi + S_{zx}\left(\sin\Phi\sin\Theta\sin\Psi + \cos\Phi\cos\Psi\right) + \\ &\quad + S_{xy}\left(\cos\Phi\sin\Theta\sin\Psi - \sin\Phi\cos\Psi\right); \\ S_V\big|_{xy} &= -S_{yz}\sin\Theta + S_{zx}\sin\Phi\cos\Theta + S_{xy}\cos\Phi\cos\Theta \end{aligned} \qquad (11)$$

corresponding to the order of the main rotating axes (*x*, *y*, *z*).
It can be seen that the variations of magnitudes of the projected aerodynamic surfaces appears in direct connection with the magnitudes of rotation angles around the three axes. If these rotations are simultaneous, theifr determinations are very difficult to do. So, is much easier to use and control the the rotation of the flying machine with small angles (is necessary that the Ψ, Θ and Φ angles to be less than 6°). The flights with greater angles are not tippical for usual flying machine except those special designed for greater maneouver angles (as space shatle).
To dimish or, contrary, to amplify the effect of the variation of virtual surface because of the rotation of flying machine around referential axes it is necessary to control its position with command surfaces separately on rolling, on pitch and on true bearing so that to obtain the desire stabilization or maneuver for the flying machine studied.

3. A NEW GENERAL DETERMINATION METHOD OF SWEPT-BACK WING

As already had been shown, the most used forms in plane of aerodynamic surfaces are those generated through streight lines. These give the advantages to easy realise constructions in a given domain of precision.

For subsonic compressible and transonic flying regimes, the sweepback wing (Fig. 8) is a very effective mean to assure the delay of appearance of critical phenomena. So that will be obtained a better response of aerodynamic surfaces (Karman, 1941).

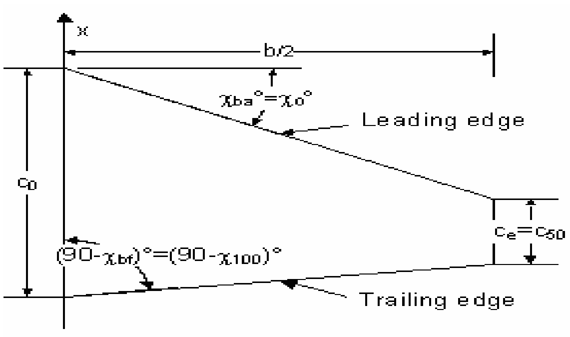

Fig.6. Topview of a wing (plan form)

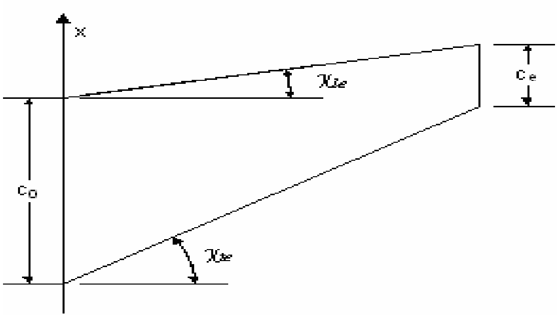

Fig.7. Forward swept wing

As shown in Fig. 8, the sweepback angle of leading edge as a function of critical Mach number (M_{cr}):

$$M_{cr}=\frac{(M_{cr})_{\chi=0}}{\cos(\chi)} \text{ or}$$

$$\chi_0=\arccos\left[\frac{(M_{cr})_{\chi=0}}{M_\infty}\right]. \qquad (12)$$

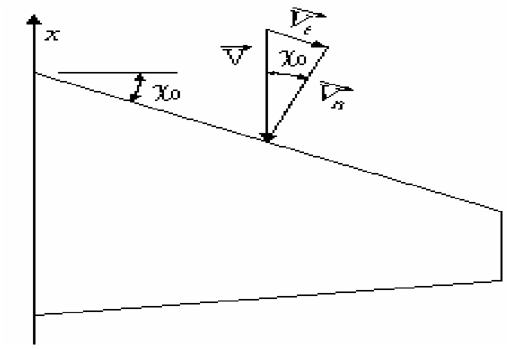

Fig.8. Critical speed formation

In the case of missiles haveing supersonic flying speed, the sweepback of the wings, empennages or ailerons with round leading edge provides the dimishing of the drug force for $M<1/\cos(\chi_0)$. But, χ_0 must not exceed the value of $70^\circ 31' 43''$, keeping enough efficiency of the aerodynamic surfaces.

The method which is shown in literature for this degtermination is based on fact that the air starts to flow on aerodynamic surface from leading edge to trailing edge. So, the decomposition of the general speed vector of the air flow, that means the speed of flying machine, is considered on leading edge. The normal and tangential components will determine the sweepback angle of the aerodynamic surface.

This new method is based on the observation that for a given general flow velocity around (on) considered aerodynamic surface (wing, aileron or empennage) the critic flow will appears on aerodynamic surface. So, the sweepback angles of the aerodynamic surface will be change too.

In what will follow a new method for determination of sweepback angle of the leading and trailing edge is shown considering the appearance of supersonic flow on profile, in subsonic compresible and transonic flight. For simple geometric relations, for given $r=c_e/c_0$, $0\le r\le 1$, the following relations beetween sweepback angles of the wing, χ_0, χ_{100} and $\chi_{\varepsilon\max}$, will be established for maxim thickness line, considering repositions of the velocities (Fig. 9):

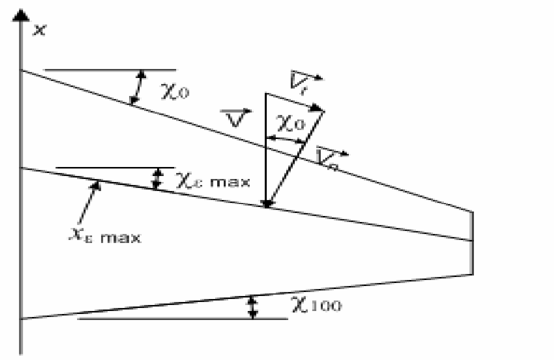

Fig.9. Calculus method

$$\tan(\chi_0)+\tan(\chi_{100})=2\frac{c_0}{b}(1-r); \qquad (13)$$

$$\tan(\chi_0)+\tan(\chi_{100})=4\frac{S_v}{b^2}\frac{1-r}{1+r}; \qquad (14)$$

$$\tan(\chi_0)+\tan(\chi_{100})=\frac{4}{\lambda}\frac{1-r}{1+r}. \qquad (15)$$

Because the aerodynamic sections don't have the same coordinate for maximum thickness, $x_{\varepsilon\max}$, the sweepback angle determined as a function of the angle of the maximum thickness line, $\chi_{\varepsilon\max}$, for a given M_{cr}, wiil be:

$$\chi_0 = \arctan\left[\tan\left(\chi_{\varepsilon\max}\right) - 2\frac{c_0}{b}\left(1-r\right)x_{\varepsilon\max}\right] \quad (16)$$

$$\chi_0 = \arctan\left[\tan\left(\chi_{\varepsilon\max}\right) - 4\frac{S_{ar}}{b^2}\frac{1-r}{1+r}x_{\varepsilon\max}\right] \quad (17)$$

$$\chi_0 = \arctan\left[\tan\left(\chi_{\varepsilon\max}\right) - \frac{4}{\lambda}\frac{1-r}{1+r}x_{\varepsilon\max}\right] \quad (18)$$

irespective,

$$\chi_{100} = \arctan\left[2\frac{c_0}{b}(1-r)\left(1+x_{\varepsilon\max}\right) - \tan\left(\chi_{\varepsilon\max}\right)\right] \quad (19)$$

$$\chi_{100} = \arctan\left[4\frac{S_{ar}}{b^2}\frac{1-r}{1+r}\left(1+x_{\varepsilon\max}\right) - \tan\left(\chi_{\varepsilon\max}\right)\right] \quad (20)$$

$$\chi_{100} = \arctan\left[\frac{4}{\lambda}\frac{1-r}{1+r}\left(1+x_{\varepsilon\max}\right) - \tan\left(\chi_{\varepsilon\max}\right)\right] \quad (21)$$

As it is known, the sweepback angles depend on the thickness of the profile. For simetrical NACA airfoil sections (Abbott et al., 1945), (Riegels, 1961), the following variations are obtained for calculus conditions: r = 0,146 $\lambda = 1{,}185$ (Fig. 10 and Fig. 11).

From comparison with references data, the values obtained for sweepback angles are greater than those recomanded. These values are characteristic for supersonic flight velocity, which means they have values of 60°-70° instead of 30°-40° for the leading edge. Also, the values for sweepback trailing angle are greater too, even for thick profiles haveing a maxim relative thickness of 3% and irespective of 6%. Fewer values for this angle are obtained for greater relative thickness of the aerodynamic profile.

4. CONCLUSIONS

In the case of the initial design of the flying machines, the aerodynamic configuration represents the first step in practical appliance of the operation demands and exploitation needs.

There is a great variety of forms and aerodynamic profiles to permit the designers to obtain the necessary aerodynamic characteristics and of maneuverability to fulfill the secure flight.

The virtual aerodynamic surfaces represent a new concept which permits a complet study for design of flying machines on obtained equivalent surfaces.The virtual surfaces can be used to determine the magnitude of real surfaces in the case of rotating the flying machine around axes of the referential system. The study cannot be directly made, but maintaining constant every time one or much more parameters and varying remaining ones.

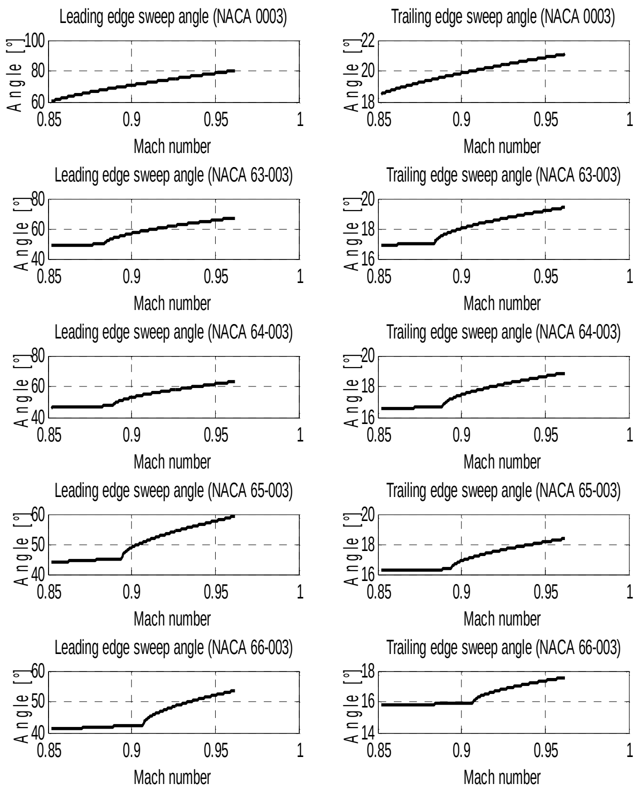

Fig.10. Sweepback angles variations for 3% relative thickness

The thin aerodynamic profiles, especially designed for great speeds, are more effective than thick ones because the critical Mach number tends to equal the relative flight speed. So, the critic flow around the profile can be easier controlled using sweepback aerodynamic surfaces.

The profiles having relative thickness of 9% are “more sensitive” because the values of critical Mach number are placed in domain 0.766 - 0.795. So, the variation of sweepback angles of the aerodynamic surfaces edges appears since the beginning of the (flight) domain considered for calculus.

But, taking into account that the shock-waves start to form and to manifest for the same relative flight speed, the influence of these is more evident in the case of these profiles.

The critical Mach number continues to diminish when relative thickness of profiles rises, but the profiles having relative thickness greater are not used for wings, empennages and/or ailerons of naval rockets.

Starting from those shown until now, it’s evident that simultaneous with diminishing the values of critical Mach number raise the influence of drag force because of earlier appearance of supersonic flow on wings, empennages and/or ailerons surfaces.

Also, it is revealed an accentuated rise of sweepback angles in comparison with the results shown in references about aircraft design for flight speed specific to the naval missiles which operate until now.

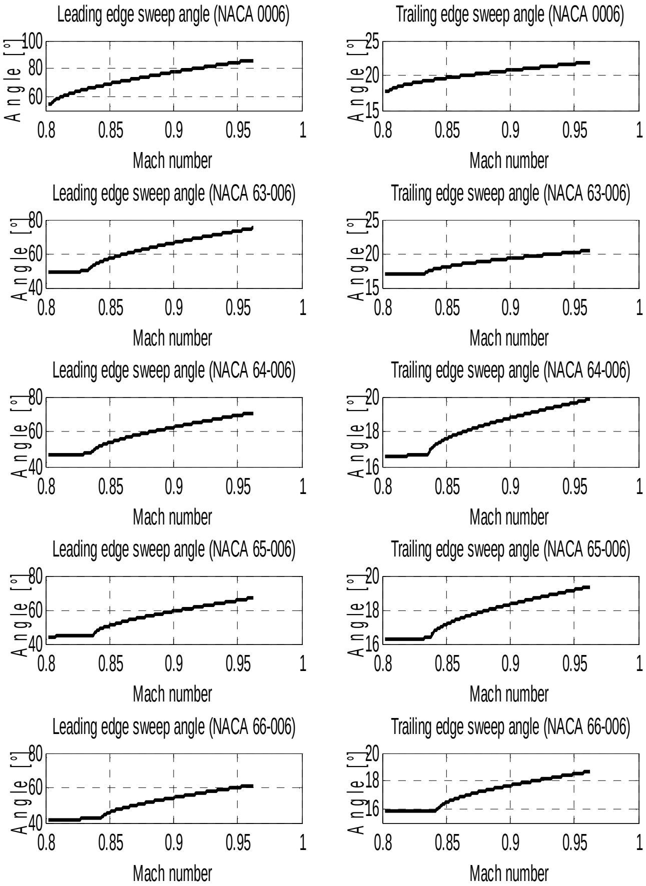

Fig.11.Sweepback angles variations for 6% relative thickness

As a first conclusion must underline that the profiles with small thickness are very useful for design and construction of wings, empennages or ailerons of naval missiles, and those having greater thickness are useful for construction of apexes or shock wave breakers.

The used method to determine sweepback angles leads to almost double values in comparison with those in speciality literature for usual flying machines. But, the authors had obtained values of sweepback angles of aerodynamic surfaces more appropriate of aerodynamic surfaces which are used for flying machine having nominal relative speed of stabil flight of 0.9 Mach.

To determine the efficient value of the sweepback angle must take in account that when this angle grows the drop of lifting force decreases, facts which have sever consequences for flight to the target.

As a general conclusion appears that the thick profiles are useful for wings, empennages and ailerons, and those with greater thickness are useful for apexes and shock wave breakers.

The reported values of sweepback angles are confirmed by the real models of naval missiles, and sweepback angle of leading edge can rich limiting value of 70°31′43″, which is characteristic for subsonic flight speed.

5. REFERENCES

1. Abbott, I.H., von Doenhoff, A.E., Stivers, L.S. (1945) - *NACA Report No. 824 - Summary of Airfoil Data*, pp. 372-386

2. AGARD-AR-318 (1995) - *Aerodynamics of 3D Aircraft Afterbodies*, NATO, pp. 143-149, ISBN 92-836-1023-7, Neuilly-sur-Seine, France

3. Anderson, J.D. (2001) - *Fundamentals of Aerodynamics*, 3rd Edition, pp. 398-408, 604-637, McGraw-Hill, ISBN 0-07-237335-0, New York

4. Cook, M.V. (2007) - *Flight Dynamics Principles*, 2nd Edition, pp. 15-30, Butterworth-Heinemann, ISBN 978-0-7506-6927-6, Oxford

5. Dingle, L., Tooley, M. (2005) - *Aircraft Engineering Principles*, pp. 541-593, Butterworth Heinemann, ISBN 0-7506-5015-X, Oxford

6. Etkin, B. (1959) - *Dynamics of Flight: Stability and Control*, pp. 13-51, John Wiley & Sons, New York

7. Etkin, B. (1972) - *Dynamics of Atmosferic Flight*, pp. 104-120, John Wiley & Sons, ISBN 0-471-24620-4, New York

8. Hull, D.G. (2007) - *Fundmentals of Airplane Flight Mechanics*, pp. 185-209, Springer-Verlag, ISBN-10 3-540-46571-5, Berlin

9. Krasnov, N.F. (1985) - *Aerodynamics*, Vol. 2, *Methods of Aerodynamic Calculations*, pp. 128-243, Mir Publishers, Moscow

10. Kuethe, A.M., Chuen-Yen, C. (1976) – *Foundations of Aerodynamics: Bases of Aerodynamics Design*, 3rd Edition, pp. 146-176, John Wiley & Sons, New York

11. Milne-Thomson, L.M. (1966) - *Theoretical Aerodynamics*, 4th Edition, pp. 270-288, Dover Publications, New York

12. Nielsen, J.N. (1960) - *Missile Aerodynamics*, pp. 112-143, 208-260, McGraw-Hill, New York

13. Riegels, F.W. (1961) - *Airfoil Sections, Results from Wind-tunnel Investigations*, pp. 103-118, 197-273, Butterworth, London

14. Siouris, G.M. (2004) - *Missile Guidance and Control Systems*, pp. 53-91, Springer-Verlag, ISBN 0-387-00726-1, New York

15. Smetana, F.O. (1997) - *Introductory Aerodynamics and Hydrodynamics of Wings and Bodies*, pp. 131-159, AIAA, ISBN 1-56347-242-2, Reston, VA

16. Torenbeek, E. (1976) - *Synthesis of Subsonic Airplane Design*, pp. 27-59, Delft University, ISBN 90-298-2505-7, Rotterdam

Received: 1 April 2010 / Accepted: November 20, 2010 / Abstract published online: December 5, 2010