ANALYTICAL METHOD OF RESIDUAL STRESS EVOLUTION INTO SUPERFICIAL LAYERS AT LOW PLASTICITY BURNISHING (LPB)

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Abstract: Mechanical surfaces enhancement technologies induce residual stresses into superficial layers of parts. Many actual researches are targeted to establish prediction methods of residual stresses evolution with the goal to mitigate the necessity to made experiments when is needed to use different methods of surfaces enhancement. We have applied a specific configuration of basic idealized mechanical units to model the inelastic material behavior in the low burnishing process. The model and the associate simulator permit to appreciate the manner of influence of technological factors on the stress evolution in the superficial layer.

Key words: Low plasticity burnishing (LPB), Modelling, Simulation

1. INTRODUCTION

Mechanical surfaces enhancement technologies (shock penning-SP; laser shock penning-LSP or low plasticity burnishing-LPB) induce residual stresses into superficial layers of parts. It is generally accepted that residual stress into superficial layers may enhances high cycle fatigue (HCF) of parts. In particularly cases residual stresses into superficial layers may enhance foreign object damage (FOD) or corrosion resistance (Prevéy, 2000; Prevéy& Cammett, 2004; Bozdana, 2005; Hsiang&Lin, 2002).

Tacked in account the role that residual stresses can have in the comportment of parts belong life cycle, the actual research are targeted to establish prediction methods of residual stresses evolution with the goal to mitigate the necessity to made experiments when is needed to use different methods of surfaces enhancement (Terzopoulos&Fleiser, 1988; Yen et al. 2005; Boyce, 2001; Belozorev et al.,2006). The limits of these methods (or models) are that there are take in account a small number of factors which defines/influences the process.

By the proposed model in this paper (an analytical model) we extend the factors space tacked into account in the residual stresses evolution prediction in low plasticity burnishing.

2. MODEL DESCRIPTION

In the low plasticity burnishing process interact the following: the material, the tool, the machine (through technological work parameters as number of rotations, feed, forces etc.). All of these elements are important to be represented into model.

2.1 The material modeling

The basic inelastic behaviors may be modeled in terms of assemblies of idealized mechanical units (Terzopoulos&Fleischer, 1988; Christensen, 1982; Mendelson, 1968; Steck et al.,2001; Malkin, 1994; Doyle, 2004; Yu et al.,2006). The ideal linear elastic unit is the spring which satisfies Hook's law:

$$\varepsilon_1 = \frac{\sigma_f}{E} \qquad (1)$$

The ideal linear viscous unit is the dashpot. The rate of increase in elongation (or contraction) is proportional to applied force:

$$\dot{\varepsilon}_2 = \frac{\sigma_f}{\lambda_2} \qquad (2)$$

The ideal plastic unit is the slip unit which is capable of arbitrary elongation or contraction when the applied force exceeds a yield force. During plastic yield, the apparent instantaneous elastic constants of the material are smaller than in elastic state. Viscoplasticity can be modeled by assembling dashpots with plastic units and is a generalization of plasticity and viscosity:

$$\dot{\varepsilon}_3 = \frac{\sigma_f - \sigma_c}{\lambda_3} \qquad (3)$$

Elastoplasticity generalizes elasticity and plasticity and may be modeled assembling elastic units with plastic units. To model the material behavior in the low plasticity burnishing process we propose a specific configuration of basic units as presented in figure 1. It is assumed that these three cells of the model reproduce the phenomena in the superficial layer, produced by burnishing, where elastic deformation, plastic deformation and creep phenomena coexist.

On the figure 1 the rolls of the burnishing head are numbered by 1, the graphical representation of the model is numbered by 2, the part is numbered by 3.

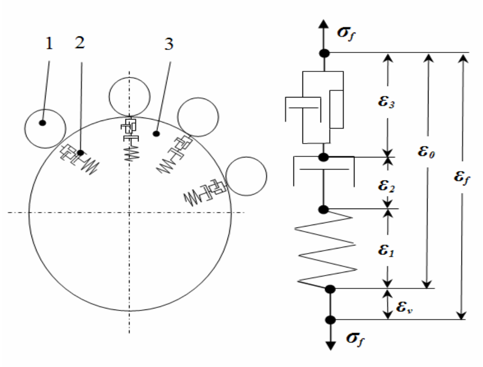

Fig.1 Inelastic deformation physical model

2.2 The tool modelling

The toll is a rolling rigid head with a number z of rolls, as in figure 2. Each part of the work piece (given by the model on figure 1) can be considered to be excited periodically by one of the rolls of the rolling head. Thus the presence of the tool upon the material can be modeled using a perturbation stress given by the relation (4).

$$\sigma_v = E\varepsilon_v = EAsin(\omega t) \tag{4}$$

2.3 The machine modelling

The presence of the machine tool in the model can be defined using the A and ω from the (4) relation parameters according to the machining parameters: number of rotations (n), feed (s), rolling time (t), number of rolls (z), grip (Δd).

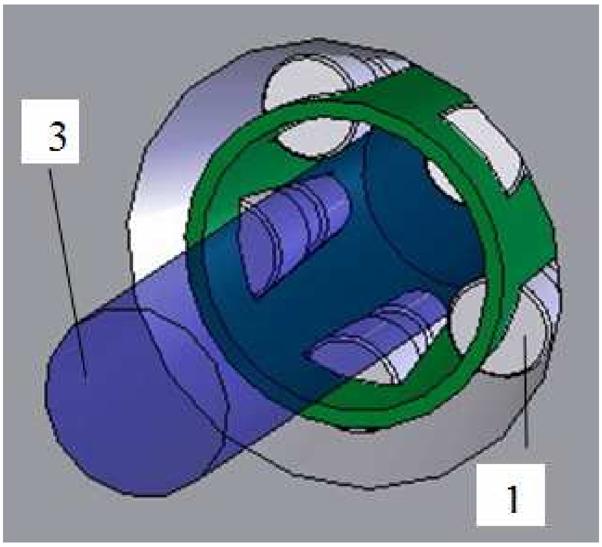

Fig.2 Burnishing tool (rolling rigid head)

2.4 Development of the model

As on figure 1 is presented, the final deformation and stress of the model are:

$$\varepsilon_f = \varepsilon_0 + \varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{5}$$

$$\sigma_f = \sigma_t + \sigma_v = \sigma_t + EAsin(\omega t) \tag{6}$$

The deformation ε_1 may be write as in (7) relation:

$$\varepsilon_1 = \frac{\sigma_t + EAsin(\omega t)}{E} \tag{7}$$

Differentiating the deformations with respect to time, results:

$$\dot{\varepsilon}_1 = \frac{\dot{\sigma}_t}{E} + A\omega cos(\omega t) \tag{8}$$

$$\dot{\varepsilon}_2 = \frac{\sigma_f}{\lambda_2} = \frac{\sigma_t + EAsin(\omega t)}{\lambda_2} \tag{9}$$

$$\dot{\varepsilon}_3 = \frac{\sigma_t + EAsin(\omega t) - \sigma_c}{\lambda_3} \tag{10}$$

$$\dot{\varepsilon}_v = \omega Acos(\omega t) \tag{11}$$

$$\dot{\varepsilon}_f = \dot{\varepsilon}_0 + \dot{\varepsilon}_v = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 \tag{12}$$

Assigning in relation (12) the above values of deformation rate, results:

$$\begin{aligned} &\omega Acos(\omega t) = \frac{\dot{\sigma}_t}{E} + A\omega cos(\omega t) + \\ &+ \frac{\sigma_t + EAsin(\omega t)}{\lambda_2} + \frac{\sigma_t + EAsin(\omega t) - \sigma_c}{\lambda_3} \end{aligned} \tag{13}$$

By simplifying of the relation (13) is obtained:

$$\begin{aligned} &\sigma'(t) + (E/\lambda_2 + E/\lambda_3)\sigma(t) + \\ &+ AE\,(E/\lambda_2 + E/\lambda_3)sin(\omega t) - \sigma_c E/\lambda_3 = 0 \end{aligned} \tag{14}$$

Solving the differential equation (14) the solution is:

$$\begin{aligned} &\sigma(t) = e^{-\frac{tE(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}} C_1 + \frac{\lambda_2\sigma_c}{(\lambda_2+\lambda_3)} + \\ &+ \frac{AE^2\lambda_2\lambda_3(\lambda_2+\lambda_3)^2\omega cos(\omega t)}{(\lambda_2+\lambda_3)[E^2(\lambda_2+\lambda_3)^2 + \lambda_2^2\lambda_3^2\omega^2]} - \\ &- \frac{AE^3(\lambda_2+\lambda_3)^3 sin(\omega t)}{(\lambda_2+\lambda_3)[E^2(\lambda_2+\lambda_3)^2 + \lambda_2^2\lambda_3^2\omega^2]} \end{aligned} \tag{15}$$

Using the initial conditions (16) the expression for C_1 is given in relation (17).

$$t = 0, \sigma[t] = 0 \tag{16}$$

$$C_1 = -\frac{\lambda_2[\lambda_2^2\lambda_3^2\sigma_c\omega^2 + E^2(\lambda_2+\lambda_3)^2(\sigma_c + A\lambda_3\omega)]}{(\lambda_2+\lambda_3)[E^2(\lambda_2+\lambda_3)^2 + \lambda_2^2\lambda_3^2\omega^2]} \tag{17}$$

With the value of C_1 from (17) the equation of σ(t) is

$$\sigma(t)=\frac{1}{(\lambda_2+\lambda_3)[E^2(\lambda_2+\lambda_3)^2+\lambda_2^2\lambda_3^2\omega^2]}.$$
$$\{\lambda_2\sigma_c[E^2(\lambda_2+\lambda_3)^2+\lambda_2^2\lambda_3^2\omega^2]-\lambda_2 e^{-\frac{tE(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}}.$$
$$[\lambda_2^2\lambda_3^2\sigma_c\omega^2+E^2(\lambda_2+\lambda_3)^2(\sigma_c+A\lambda_3\omega)]+ \qquad (18)$$
$$+AE^2\lambda_2\lambda_3(\lambda_2+\lambda_3)^2\omega cos(\omega t)-$$
$$-AE^3(\lambda_2+\lambda_3)^3 sin(\omega t)\}$$

Considering an integer number of periods as Sin(ωt)=0 and Cos(ωt)=1 the result is:

$$\sigma(t)=\frac{\lambda_2 e^{-\frac{tE(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}}(e^{-\frac{tE(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}}-1)}{(\lambda_2+\lambda_3)[E^2(\lambda_2+\lambda_3)^2+\lambda_2^2\lambda_3^2\omega^2]}. \qquad (19)$$
$$.[\lambda_2^2\lambda_3^2\sigma_c\omega^2+E^2(\lambda_2+\lambda_3)^2(\sigma_c+A\lambda_3\omega)]$$

To introduce technological parameters of the process we use the relations (20, (21) where *d* is part diameter, Δ*d* is variation of part diameter by burnishing, *z* is number of rolls on burnishing head, *n* is part revolution on minute.

$$A=(d+2\Delta d)/d \qquad (20)$$

$$\omega=(2\pi nz)/60 \qquad (21)$$

Equation (19), considering relations (20) and (21), can be written as :

$$\sigma(t)=\frac{\lambda_2 e^{-\frac{tE(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}}(e^{-\frac{tE(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}}-1)}{(\lambda_2+\lambda_3)[0.01n^2z^2\lambda_2^2\lambda_3^2+E^2(\lambda_2+\lambda_3)^2]}.$$
$$.\{0.01n^2z^2\lambda_2^2\lambda_3^2\sigma_c+E^2(\lambda_2+\lambda_3)^2. \qquad (22)$$
$$.[\frac{0.1nz(d+2\Delta d)\lambda_3}{d}+\sigma_c]\}$$

The relation (22) represents the model of stress evolution into superficial layers at low plasticity burnishing taken in account, both, the material and process characteristics.

3. SIMULATION AND DISCUTION

On the model presented on relation (22) is difficult to analyze and to made relevance for the influence of every factor involved into process. On an other hand between factors may exist synergy which can influence the process in a complex way.

To gain more relevance in what is the influence of every factors to the process, a simulator was build.

The simulator is build on Mathematica 6 environment (Wellin et al., 2005). The output get from model evaluation is an interactive object, figure 3, containing more controls (sliders) that may be used to vary the value of one or more parameters. In the case of multiple variables/factors and when it is want to see the effect of changing all of them, simultaneously, the simulator solves this problem by the autorun feature which control that runs all the variables through their ranges of values. Using this simulator, there are two ways to made simulations. A first one is bookmarking combinations of parameters values to find a needle in a haystack as a particular combination of multiple parameter values that yields a particularly interesting result. The second way is design of experiments which permits understanding of the effect that different parameters have on performance and to focus on controlling variation on the critical dimensions.

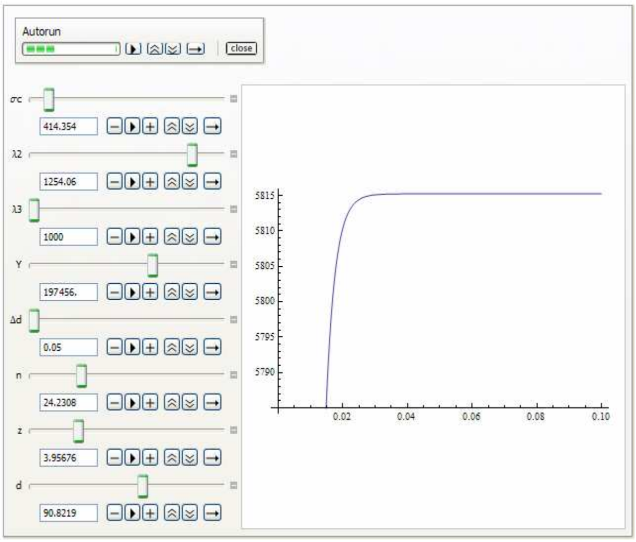
σc 414.354
λ2 1254.06
λ3 1000
Y 197456.
Δd 0.05
n 24.2308
z 3.95676
d 90.8219

Fig.3 Simulator for the inelastic deformation model

Table1. Experimental plan to simulate the model

Nr.	σ_c	λ_2	λ_3	Δd	n	z	d	σ(t)
1	1	1	1	1	1	1	1	271
2	1	1	1	1	2	2	2	338
3	1	1	1	1	3	3	3	462
4	1	2	2	2	1	1	1	275
5	1	2	2	2	2	2	2	376
6	1	2	2	2	3	3	3	564
7	1	3	3	3	1	1	1	278
8	1	3	3	3	2	2	2	431
9	1	3	3	3	3	3	3	670
10	2	1	2	3	1	2	3	292
11	2	1	2	3	2	3	1	426
12	2	1	2	3	3	1	2	393
13	2	2	3	1	1	2	3	310
14	2	2	3	1	2	3	1	473
15	2	2	3	1	3	1	2	445
16	2	3	1	2	1	2	3	402
17	2	3	1	2	2	3	1	520
18	2	3	1	2	3	1	2	495
19	3	1	3	2	1	3	2	308
20	3	1	3	2	2	1	3	356
21	3	1	3	2	3	2	1	500
22	3	2	1	3	1	3	2	448
23	3	2	1	3	2	1	3	487
24	3	2	1	3	3	2	1	604
25	3	3	2	1	1	3	2	441
26	3	3	2	1	2	1	3	497
27	3	3	2	1	3	2	1	649

The second way, design of experiments, was used.

To analyze the influence of different factors involved in process, an experimental plan was applied. For every factor was taken in account three levels. An Taguchi orthogonal array L27(3**7) was chosen. The experimental plan is presented in table 1. An orthogonal array means the design is balanced so that factor levels are weighted equally. Each factor can be evaluated independently of all the other factors, so the effect of one factor does not influence the estimation of another factor. Each combination of control factor levels is called a run. The Taguchi design provides the specifications for each experimental test run. Responses are measured at selected combinations of the control factor levels.

The influence of factors involved in process is presented in figure 4.

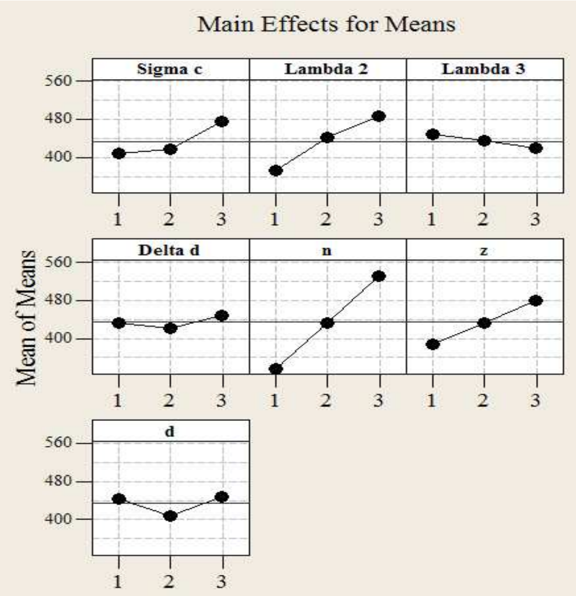

Fig.4 Effects of factors acting into model

In many process applications, potentially influential variables are numerous. Screening reduces the number of variables by identifying the key variables that affect the result. This reduction allows to focus process improvement efforts on the really important variables. Screening may also suggest the "best" or optimal settings for these factors, and indicate whether or not curvature exists in the responses.

4. CONCLUSIONS

We have applied a specific configuration of basic idealized mechanical units to model the inelastic material behavior in the low burnishing process. The model and the associate simulator permit to appreciate the manner of influence of technological factors on the stress evolution in the superficial layer. The adjustment of the diameter diminishment (Δd) is done in function of the desired purpose, taking in consideration the relative position of the rolls towards the part axis. The adjustment of the processing work settings (n, z) is done taking in consideration the influence that each of these factors have on the residual stress. We make no particular attempts to model specific materials accurately and these may be a direction for new development.

5. REFERENCES

1.Belozorev V.V.,Makhatilova A.I., Sobotina V.V., (2006), *Cyclic strength of parts with overlappig zones formed by roller burnishing*, Strength of Materials, Volume 38, pp. 324-327.

2.Boyce B.L., (2001). *The Residual Stress State Due to a Spherical Hard-Body Impact*, *Mechanics of Materials*, Vol. 33,pp. 441-454.

3.Bozdana T.A, (2005), *On the mechanical surface enhancement techniques in aerospace industy - a review of technology*, Journal of Aircraft Engineering and Aerospace Technology, Vol.77, pp. 279-292.

4.Christensen R.M, (1982), *Theory of viscoelasticity*, 2nd Academic Press Publishing House, New York

5.Doyle J.F., (2004). *Modern Experimental Stress Analysis*, John Wiley & Sons, England.

6.Hsiang S.H., Lin S.L., (2002), *An Investigation of the Process Parameters for Calibre Rolling Using a Complex Analytical Model*, The International Journal of Advanced Manufacturing Technology, Volume 19, pp. 343-350.

7.Malkin A.Ya., (1994), *Rheology Fundamentals*, ChemTec Publishing, Toronto.

8.Mendelson A.,(1968), *Plasticity-Theory and Application*, Mac-millan, New York.

9.Prevéy P.S., *(2000)*, *The Effect of Cold Work on the Thermal Stability of Residual Compression in Surface Enhancement IN718*, Procedings 20th ASM Materials Solutions Conference & Exposition, St.Louis, Missouri, pp. 255-264.

10.Prevéy P.S, Cammett J.T., (2004), *The influence of surface enhancement by low plasticity burnishing on the corrosion fatigue performance of AA 7075-T6*, International Journal of Fatigue, Vol.269, pp. 975-982.

11.Steck E., Ritter R., Pfeil U., Ziegenbein A., (2001), *Plasticity of Metals: Experiments, Models, Computation*, WILEY-VCH Verlag GmbH,Bonn.

12.Terzopoulos D., Fleischer K., (1988), *Modeling Inelastic Deformation Viscoelasticity, Plasticity, Fracture*, Computer Graphics, Volume 22, Number 4, pp. 269-278.

13.Wellin P., Kamin S., Gaylord R., (2005) *An Introduction to Programming with Mathematica*, Cambridge University Press, Cambridge, UK.

14.Yen Y.C., Sartkulvanish P., Altan T., (2005), *Prediction of surface Finish and Residual Stress in Hard Roller Burnishing with FE Modeling*, Annals of the CIRP, Vol. 54, pp. 237-242.

15.Yu M.H., Ma G.W., Qiang H.F., Zhang Y.Q, (2006), *Generalized Plasticity*, Springer-Verlag Berlin Heidelberg

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